Optimal Preventive Maintenance Schedule based on Lifecycle Cost and Time-Dependent Reliability

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ABSTRACT

Reliability is an important engineering requirement for consistently delivering acceptable product performance through time. It also affects the scheduling for preventive maintenance. Reliability usually degrades with time increasing therefore, the lifecycle cost due to more frequent failures which result in increased warranty costs, costly repairs and loss of market share. In a lifecycle cost based design, we must account for product quality and preventive maintenance using time-dependent reliability. Quality is a measure of our confidence that the product conforms to specifications as it leaves the factory. For a repairable system, preventive maintenance is scheduled to avoid failures, unnecessary production loss and safety violations. This article proposes a methodology to obtain the optimal scheduling for preventive maintenance using time-dependent reliability principles. An optimization algorithm maximizes the time for preventive maintenance by improving the system reliability, so that the lifecycle cost stays below a specified target. The lifecycle cost includes a production, an inspection, and an expected variable cost. All costs depend on quality and/or reliability. Preventive maintenance is performed at the time when the improved reliability falls below an acceptable reliability target. The methodology also identifies the most critical component(s), or failure modes, which if repaired, will improve the system reliability the most. We demonstrate the proposed approach using an automotive roller clutch example.

1. INTRODUCTION

Customers and product manufacturers demand continued functionality of complex equipment and processes. Degradation of material properties in time and stochastic operating conditions such as loading, result in a lowered resale value due to inconsistent performance and

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14. ABSTRACT

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even premature failure, increased warranty cost and customer annoyance. In product design, it is desirable to consider the attributes of survivability, quality, reliability, maintainability, and safety, and reduce scrap, rework, and maintenance time. This can be achieved by selecting dimensions and materials so that the system performance satisfies all design specifications over the planned life-time. An important factor in a customer's buying decision is the total lifecycle cost which can be affected considerably by a high maintenance cost.

Quality and reliability are two important attributes that must be addressed simultaneously. Quality measures the conformance to design specifications as the product leaves the factory to begin its lifecycle. In this work, quality is quantified by the probability that the performance measures meet the specifications at the initial time of its lifecycle; i.e. right after it is produced. Reliability is defined as the probability that the system will perform its intended function for a specified interval of time, under stated operating and environmental conditions. Reliability is, therefore, related to product functionality which is determined by the so-called "hard" and "soft" failures [1, 2]. In a hard failure the system loses functionality due to a complete breakdown, while in a soft failure the system is functional but one or more performance measures are out of conformance.

Maintenance is also an important attribute. It involves any activity that can restore or recover a system should it breaks down or gets out of order. This type of maintenance, known as repair or unscheduled maintenance, can be very costly and should be avoided. In contrast, a scheduled maintenance occurs before any failure or malfunction in order to keep the system in working order preventing a potentially serious damage. Preventive maintenance is therefore, a schedule of planned maintenance actions performed in an attempt to avoid failures, unnecessary production loss and safety violations.

In preventive maintenance, when the maintenance frequency increases, the deterioration failure rate is reduced [3]. As a system cannot be maintained at all times, a way is needed to decide when inspection/maintenance is needed. To avoid unnecessary maintenance, which increases the cost and may also increase the risk of further wear, the effectiveness of a preventive maintenance schedule should depend on quality, reliability and a lifecycle cost analysis. Barlow and Hunter [4] did a pioneering work on optimum preventive maintenance policies. Since then, many works have been done along this line. Survey papers such as in [3-8] cover the subject of optimal maintenance.

In our previous work [9], a design methodology was presented based on the product lifecycle cost considering uncertainty and variability through time-dependent reliability in order to simultaneously meet the needs of the manufacturer (low production, inspection and warranty costs) and the customer (customer satisfaction and better resale value). Although design under uncertainty, or reliability-based design (RBD), has been extensively studied [10-14], RBD considering the lifecycle cost has not attracted much attention due to the complexity and high computational effort in calculating the time-dependent reliability. Only a limited number of studies exist which mainly use simplifying assumptions in the calculation of time-dependent reliability [1, 2, 15-17] or use lifecycle cost in design to account for reliability, maintenance and warranty [18-20].

The intent of this paper is to increase the time before preventive maintenance is needed using design optimization, considering time-dependent phenomena such as degradation of certain components. Consider a system with multiple responses which must conform to certain specification limits set by the designer. For demonstration purposes, degradation in time changes

the statistics of the inputs (e.g. operating loads, material properties, etc.) and the resulting statistics of the responses. Fig. 1 shows a probabilistic view of this process over time.
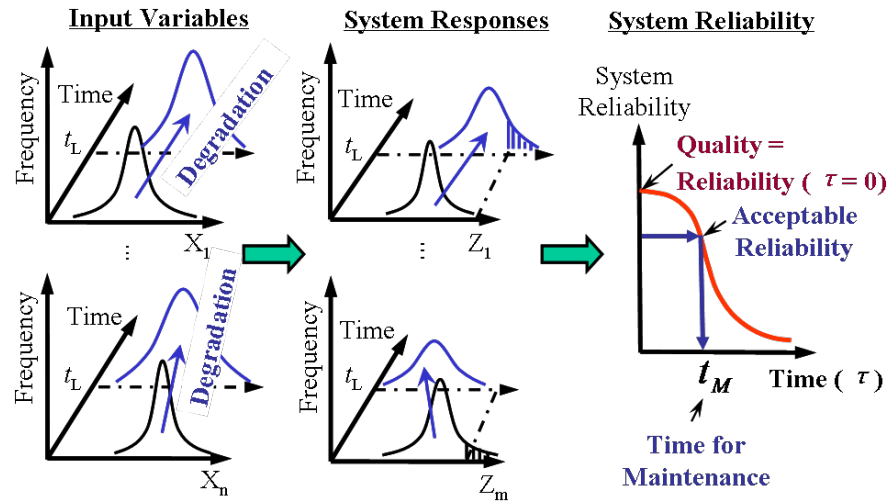

Figure 1.   Uncertainty propagation for time-dependent system reliability (modified from Savage et al [1])

At the left of Fig. 1, the probability density functions of the input random variables X_i change over time due to degradation. They are therefore, denoted by $X_i(\tau)$ to indicate that they are time-dependent. The designer may control only the initial probability density functions. The system response distributions also change over time due to the changes in the inputs. It is assumed that each response $Z_j(\tau)$ is a function of the inputs $X_1(\tau), X_2(\tau), ..., X_n(\tau)$. The system reliability versus time is shown at the right of Fig. 1. It is built over time by calculating the system time-dependent reliability or probability of conformance. As a special case, quality can be viewed as the initial reliability at $\tau = 0$. If an acceptable reliability level is prescribed, the time to maintenance t_M can be determined. The efficient estimation of the time dependent reliability is thus the key to determining the time to maintenance. Maximizing the time to maintenance for a given level of system reliability is the goal of this research.

To date, few researchers have proposed methods to estimate the system reliability by discretizing time and using time invariant limit-state methods. Using a different approach, Son and Savage [21] tracked time-variant, limit-state surfaces in the standard normal space. Incremental failure probabilities were evaluated and summed at discrete time intervals to estimate the mean time to soft failure (MTTSF) and its variance. It is easier, however, for managers and engineers to understand monetary measures rather than the traditional reliability measures. Thus, monetary measures are commonly used as optimization objectives or constraints. The costs of degrading systems can be used to determine the means and tolerances.

With respect to performance reliability, there have been a few research activities in which the present value (or worth) of expected quality losses are used. The latter are measures of non-conformance of the responses with respect to desired specifications. Chou et al. [22, 23] augmented present value with the production cost C_P to form a total cost, and then performed

tolerance allocation by formulating an optimization problem. Son et al. [24] used a probabilistic expected loss of quality and extended its application to degrading mechanistic systems with multiple target/smaller/larger-is-best performance metrics. The probabilistic measure was optimized using aspects of Reliability-Based Design Optimization (RBDO) methods.

In our previous research [9], we used a time-variant reliability analysis and loss of quality costs [2] in the design of multi-response systems. The optimal design was calculated by minimizing the lifecycle cost while satisfying quality and reliability constraints throughout the lifecycle of the system. The conformance of multiple responses was treated in a series-system fashion. In this research, we maximize the *first* time for preventive maintenance subject to an acceptable quality, time-dependent reliability and cost constraints. An automotive roller clutch example illustrates our approach and provides details on the usefulness and practicality of the proposed methodology for optimal preventive maintenance scheduling.

The paper is organized as follows. After a brief introduction to time-dependent reliability analysis in Section 2, the definition of lifecycle cost and its calculation are provided in Section 3. Section 4 presents the optimization problem for maximizing the time to maintenance and Section 5 uses an automotive roller clutch example to demonstrate the benefits of the proposed methodology. Finally, a summary and conclusions are provided in Section 6.

2. RELIABILITY ANALYSIS

In reliability analysis, we estimate the probability of failure due to variability and/or uncertainty using a probabilistic quantification of the variation/uncertainty and a definition of failure. In time-independent (or time-invariant) reliability analysis, the uncertainty is quantified using random variables. A time-independent limit state function $g(\mathbf{X})$ is used, where the vector $\mathbf{X}$ represents the input random variables with a joint probability density function $f_{\mathbf{X}}(\mathbf{x})$. The probability of failure is given by

$$P_f = P\big(g(\mathbf{X}) < 0\big) = \int \cdots \int_{g(\mathbf{x})<0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}. \tag{1}$$

Because the multi-dimensional integral of Eq. (1) is difficult to evaluate, various analytical and simulation-based methods have been developed [10-14, 25-30].

2.1 TIME-DEPENDENT RELIABILITY ESTIMATION

In many engineering systems, the system response (output) depends on time and is described by a random process $g(\mathbf{d}, \mathbf{X}, \tau)$. For each realization of the input deterministic variables $\mathbf{d}$ and random variables $\mathbf{X}$, the output is a time-dependent sample function. Denoting the time interval of interest by $[t_{\min}, t_{\max}]$, the probability of failure within this time interval is defined as

$$P_f^c(\mathbf{d}, \mathbf{X}; t_{\min}, t_{\max}) = P\big(\exists t \in [t_{\min}, t_{\max}], g(\mathbf{d}, \mathbf{X}, t) < 0\big), \tag{2a}$$

or

$$F^c(\mathbf{d}, \mathbf{X}, t) = P_f^c(\mathbf{d}, \mathbf{X}; 0, t) = P\big(\exists \tau \in [0, t], \, g(\mathbf{d}, \mathbf{X}, \tau) < 0\big) \tag{2b}$$

where F^c denotes what is known as *cumulative* probability of failure [30], indicating that failure occurs if the limit state function becomes negative at least at one instance within the time interval. In general, the definition of the *cumulative* probability of failure in Eq. (2) is different from the *instantaneous* probability of failure

$$P_f^i(\mathbf{d}, \mathbf{X}, t) = P(g(\mathbf{d}, \mathbf{X}, t) < 0), \tag{3}$$

which is calculated in a quasi-steady sense by fixing time t and replacing the random process by a random variable. As a result, it does not account for possible failures before time t. The conceptual difference between the cumulative and the instantaneous probabilities of failure is very important.

In our previous work [9], we gave a brief literature review of time-dependent reliability evaluation methods, and presented a composite limit state concept to ease the estimation of the time-dependent reliability in Eq. (2). The composite limit state in the time period $[0, t]$ is defined as

$$g^c(\mathbf{d}, \mathbf{X}, t) = \{\exists \tau \in [0, t], \, g(\mathbf{d}, \mathbf{X}, \tau) < 0\} \approx \bigcup_{l=0}^{L} \{g(\mathbf{d}, \mathbf{X}, \tau_l) \le 0\}, \tag{4}$$

where $\tau_l \in [0, t]$ for $\tau_0 = 0 < \tau_1 < \ldots < \tau_l < \ldots < \tau_L = t$. If $L \to \infty$, the approximation in Eq. (4) becomes an equality.

The event of first passage in a given time interval is equivalent to the event of the maximum response in the same time interval exceeding an allowable threshold value. The definition of the cumulative probability of failure $F^c(t)$ is thus equivalent to the probability of first passage. If the maximum response in the time period [0, t] is

$$y^{\max}(\mathbf{d}, \mathbf{X}, t) = \max_{0 \le \tau \le t} y(\mathbf{d}, \mathbf{X}, \tau), \tag{5}$$

the *time-independent* "composite" limit state for a fixed time interval [0, t] is defined as $g^c(\mathbf{d}, \mathbf{X}, t) = 1 - y^{\max}(\mathbf{d}, \mathbf{X}, t)/y^t = 0$ where y^t is the allowable threshold value. The composite failure is then indicated by

$$g^c(\mathbf{d}, \mathbf{X}, t) = 1 - y^{\max}(\mathbf{d}, \mathbf{X}, t)/y^t \le 0. \tag{6}$$

In this work, we use the composite limit state concept in the illustrative example of Section 5. We calculate the most probable points (MPP) and linearize each of the original limit states at the corresponding MPP, and then use Ditlevesen's narrow bounds [31] to estimate the cumulative probability of failure. References [9, 31] provide more details.

3. COST MODEL

The lifecycle cost C_L includes the production cost C_P, the inspection cost C_I and an expected variable cost C_V^E [2, 32]. These costs are a function of quality and reliability. The lifecycle cost is expressed as

$$C_L(\mathbf{d}, \mathbf{X}, t_f, r) = C_P(\mathbf{d}, \mathbf{X}) + C_I(\mathbf{d}, \mathbf{X}, t_0) + C_V^E(\mathbf{d}, \mathbf{X}, t_f, r), \tag{7}$$

where $\mathbf{d}$ is the vector of deterministic design variables, $\mathbf{X}$ is the vector of random design variables, t_f is the product life, and r is the interest rate to translate all future costs to a present value. Note that all components of C_L depend on $\mathbf{d}$ and $\mathbf{X}$. The inspection cost represents the cost to inspect or fix initial defects. The expected variable cost C_V^E is a function of the time-dependent reliability which is used to estimate the expected present value of repairing and/or replacing the failed products during the product lifetime.

The calculation of C_P and C_I costs is straightforward. The expected variable cost is due to loss of quality and is given by [2, 32]

$$C_V^E(t_f) = \int_0^{t_f} c_f(\tau) f_T(\tau) e^{-r\tau} d\tau \tag{8}$$

where $c_f(\tau)$ is the cost of failure at time τ, $e^{-r\tau}$ is the discount factor, and $f_T(\tau)$ is the probability density function (PDF), conditional on the event that the system is in a conforming state at the initial time $\tau = 0$. The latter is calculated as the time derivative of the cumulative probability of the failure $F^c(\mathbf{d}, \mathbf{X}, \tau)$; i.e.

$$f_T(\tau) = \frac{dF^c(\mathbf{d}, \mathbf{X}, \tau)}{d\tau}. \tag{9}$$

Clearly, $C_V^E(t_f)$ depends on the time-dependent probability of failure, which is expressed by its PDF $f_T(\tau)$. The calculation of $f_T(\tau)$ is a significant challenge and was addressed in [9].

4. PROBLEM STATEMENT

It is desirable to increase the time for inspection and maintenance of a repairable system while satisfying reliability and total cost requirements. In order to deliver "best value," an optimization must determine the optimal design by maximizing the time to maintenance while keeping the system reliability above a specified target and the lifecycle cost below an acceptable level.

In this work, we consider the first time to maintenance. The following design optimization problem is solved

$$\max_{\mathbf{d},\mathbf{\mu_X},\mathbf{\sigma_X},t_M} t_M$$

$$\text{s.t.} \quad C_L\left(\mathbf{d},\mathbf{\mu_X},\mathbf{\sigma_X},t_M,r\right)\le C_L^t \tag{10a}$$

$$F^i\left(\mathbf{d},\mathbf{X},t_0\right)\le 1-R^t\left(t_0\right) \tag{10b}$$

$$F^c\left(\mathbf{d},\mathbf{X},t_M\right)\le 1-R^t\left(t_M\right) \tag{10c}$$

$$\mathbf{d}_L \le \mathbf{d} \le \mathbf{d}_U, \tag{10d}$$

$$\mathbf{\mu_{X}}_L \le \mathbf{\mu_X} \le \mathbf{\mu_{X}}_U, \quad \mathbf{\sigma_{X}}_L \le \mathbf{\sigma_X} \le \mathbf{\sigma_{X}}_U \tag{10e}$$

where the time to preventive maintenance, t_M, is maximized by varying the deterministic design variables $\mathbf{d}$, and the means $\mathbf{\mu_X}$ and standard deviation $\mathbf{\sigma_X}$ of the random variables $\mathbf{X}$. In Eq. (10a), $C_L\left(\mathbf{d},\mathbf{\mu_X},\mathbf{\sigma_X},t_M,r\right)$ comprises the mean production cost $C_P\left(\mathbf{d},\mathbf{\mu_X}\right)$, the mean inspection cost $C_I\left(\mathbf{d},\mathbf{\mu_X},t_0\right)$, and the expected variable cost $C_V^E\left(\mathbf{d},\mathbf{\mu_X},t_M,r\right)$ up to t_M.

Eq. (10b) ensures that an acceptable initial quality is met. $F^i\left(\mathbf{d},\mathbf{X},t_0\right) = P_f^i\left(t_0\right) = P(g\left(\mathbf{d},\mathbf{X},t_0\right)<0)$ is the instantaneous CDF for quality at time $t_0 = 0$ and $R^t\left(t\right)$ is the system target reliability for $t \in \left[t_0, \, t_M\right]$. In Eq. (10c), the cumulative probability of failure $F^c\left(\mathbf{d},\mathbf{X},t\right)$ at any time $t \in \left[t_0, \, t_M\right]$ must be less that an acceptable probability of failure $\left(1-R^t\left(t_M\right)\right)$ at t_M. Eqs (10d) and, (10e) provide lower (L) and upper (U) bound constraints for $\mathbf{d}$, $\mathbf{\mu_X}$, and $\mathbf{\sigma_X}$, respectively.

5. A ROLLER CLUTCH EXAMPLE

An automotive roller clutch is used to demonstrate how the proposed formulation of Section 4 can be used in design. The model is discussed in detail in [2, 33, 34]. The clutch assembly consists of a hub, a cage, and four rollers made of 1020 steel as well as four springs (see Fig. 2). The springs push the rollers against the cage in order to remain in contact with the cage and hub. If the hub turns counter-clockwise, the rollers bind and the resulting torque "locks" the cage with the hub. Otherwise, the rollers slip and there is no torque conversion. The main features of the design include the hub and roller diameters D and d, respectively, the cage outer and inner diameters B and A, the roller length L, the number of rollers N, and the material modulus of elasticity E and Poisson's ratio v.
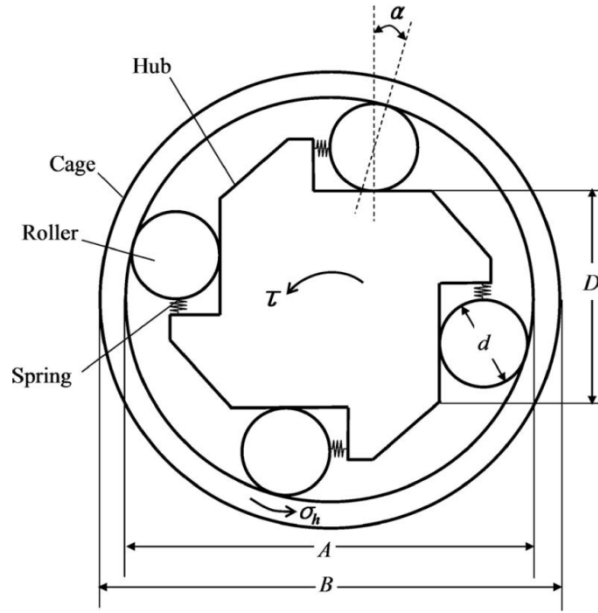
Figure 2. Roller clutch schematic

A proper operation is ensured by three performance measures: the contact angle α, the torque capacity τ, and the hoop stress σ_h. The contact angle ensures proper binding and slipping. If the value of the angle is greater than the upper specification limit or lesser than the lower specification limit, the clutch does not work correctly and it must be reworked or scrapped. The torque capacity is provided by the friction between the hub, roller and cage. The design must provide sufficient torque without exceeding material compressive strengths. The hoop stress results in from the circumferential roller friction force on the cage. It must be kept below a maximum to prevent fatigue failure in the pipe-like cage.

The angle α, torque capacity τ, and hoop stress σ_h are functions of the design variables [34], and are given by

$$\alpha = \cos^{-1}(S),$$
(11a)

$$\tau = NL\left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2 d}{4(D+d)}\sqrt{1-S^2},$$
(11b)

and

$$\sigma_h = \frac{N}{2\pi}\left(\frac{\sigma_c}{c_1}\right)^2\left(\frac{Dd}{(D+d)}\right)\frac{S}{A}\left(\frac{B^2+A^2}{B^2-A^2}\right),$$
(11c)

where

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$$S = \frac{D+d}{A-d} \tag{11d}$$

is a geometric ratio, and

$$c_1 = \frac{1}{4}\sqrt{\frac{\pi E}{2(1-v^2)}} \tag{11e}$$

is a constant, and σ_c is the average permissible contact stress.

The design variables in this work are the hub diameter D, roller diameter d, and the inner cage diameter A. The remaining design parameters in the angle, torque and stress relations are assumed constant with $L = 80$ mm, $B = 120$ mm, and $N = 4$. The properties of the selected 1020 steel material for the hub, roller, and cage are $v = 0.29$, $E = 207$ GPa, and $\sigma_c = 3,790$ MPa. The design variables are assumed to be statistically independent and normally distributed. Their means and standard deviations are assumed to be within the following bounds

$$55.0973 \le \mu_D \le 55.4973, \ 22.66 \le \mu_d \le 23.06, \ 101.49 \le \mu_A \le 101.89,$$
$$0.04 \le \sigma_D \le 0.08, \ 0.03 \le \sigma_d \le 0.1, \ 0.07 \le \sigma_A \le 0.133. \tag{12}$$

The three design variables degrade over time. The surface wear on the clutch parts causes the inner diameter A of the cage to increase over time, and the hub and roller diameters D and d to decrease over time. We assume the wear degradation rate is constant over time with a wear rate of $k = 2.5 \times 10^{-4}$ mm/year. The time dependency of the three design variables is thus expressed by

$$D(t) = D(1-kt), \ d(t) = d(1-kt), \ \text{and } A(t) = A(1+kt). \tag{13}$$

For proper operation, the angle must be 0.11 radians with a tolerance of ± 0.06 radians. Also, the torque must be at least 3000 Nm, and the hoop stress must be at most 400 MPa. The following four constraints express therefore, the performance measures

$$g_1(D,d,A) = 0.05 - \alpha \le 0 \tag{14a}$$
$$g_2(D,d,A) = \alpha - 0.17 \le 0 \tag{14b}$$
$$g_3(D,d,A) = 3000 - \tau \le 0$$

(14c)

$$g_4(D,d,A) = \sigma_h - 400 \times 10^6 \le 0 \ .$$

(14d)

The production cost is defined as [34]

$$C_P(\mathbf{\mu_X}, \mathbf{\sigma_X}) = \left(3.5 + \frac{0.75}{3\sigma_D}\right) + \left(3.0 + \frac{0.65}{3\sigma_d}\right) + \left(0.5 + \frac{0.88}{3\sigma_A}\right) \tag{15}$$

indicating that a higher precision (smaller tolerance) increases the production cost.

We assume that the lifetime is $t_f = 20$ years and the interest rate is $r = 3\%$. Similarly to [2], we define the inspection cost as $C_I(\mathbf{\mu_X}, \mathbf{\sigma_X}) = c_s F^i(\mathbf{\mu_X}, \mathbf{\sigma_X}, t_0 = 0)$ where the cost for scrap is $c_s = \$20$ and $F^i(\mathbf{\mu_X}, \mathbf{\sigma_X}, t_0 = 0) = P(\bigcup_i^4 (g_i(D, d, A, t_0) > 0))$. The expected variable cost is given by Eq. (8) as $C_V^E(t_f) = c_f \int_0^{t_f} f_T^c(t) e^{-rt} dt$, where the rework cost is assumed constant and equal to $c_f = \$20$, and the PDF $f_T^c(t)$ is the derivative of the cumulative CDF $F_T^c(t)$ with respect to time according to Eq. (9). The expected variable cost $C_V^E(t_f)$ is calculated using a trapezoidal integration scheme.

In this example, we did not use a minimum acceptable initial quality as Eq. (10b) of the general problem formulation indicates. Also, we did not have any deterministic design variables **d**. The general design optimization problem of Eq. (10) was then reduced to

$$\max_{\mathbf{\mu_X}, \mathbf{\sigma_X}, t_M} t_M$$

$$\text{s.t.} \quad C_L(\mathbf{\mu_X}, \mathbf{\sigma_X}, t_M, r) \le C_L^t \tag{16a}$$

$$F^c(\mathbf{\mu_X}, \mathbf{\sigma_X}, t_M) \le 1 - R^t(t_M) \tag{16b}$$

$$\mathbf{\mu_{X_L}} \le \mathbf{\mu_X} \le \mathbf{\mu_{X_U}}, \quad \mathbf{\sigma_{X_L}} \le \mathbf{\sigma_X} \le \mathbf{\sigma_{X_U}} \tag{16c}$$

where the cumulative failure probability of the system, given by $F^c(\mathbf{\mu_X}, \mathbf{\sigma_X}, t_M) = P\left(\bigcup_i^4 (g_i(D, d, A) < 0)\right)$, is time-dependent because D, d, and A depend on time (see Eq. 13). The normally distributed random design vector $\mathbf{X} = \{D, d, A\}$ has a mean $\mathbf{\mu_X} = \{\mu_D, \mu_d, \mu_A\}$ and a standard deviation $\mathbf{\sigma_X} = \{\sigma_D, \sigma_d, \sigma_A\}$. The random variables D, d, and A are assumed independent because the kt term in Eq. (13) is very small. The lower and upper bounds for $\mathbf{\mu_X}$ and $\mathbf{\sigma_X}$ are provided by the side constraints of relation (12), and the initial values for $\mathbf{\mu_X}$ and $\mathbf{\sigma_X}$ are {55.30, 22.86, 101.69} and {0.06, 0.065, 0.1015}, respectively. The target reliability at the time for maintenance is set to $R^t(t_M) = 0.9$. In order to study the effect of the target lifecycle cost C_L^t, we solve the problem for $C_L^t = (\$18, \$19, \ldots, \$28)$.

For $C_L^t = \$18$, an optimal design for the maximum time to maintenance $t_M = 7.01$ years is obtained at $\mu_D = 55.4946$ mm, $\mu_d = 22.7562$ mm, $\mu_A = 101.49$ mm, $\sigma_D = 0.08$ mm, $\sigma_d = 0.064$ mm, and $\sigma_A = 0.111$ mm. The hub diameter D reaches its upper bound and the cage internal diameter A reaches its lower bound. The constraint (16b) becomes active indicating that the

target reliability requirement of 0.9 is met. The four limit states corresponding to the four constraints of Eqs. (14a) through (14d) are shown in Fig. 3 where the optimal design is shown in the standard normal space for $t_0 = 0$ and $t_M = 7.01$ years by projecting the four limit states and the corresponding MPPs onto the $\{u_1 \equiv u_D \quad u_3 \equiv u_A\}$ space. The progression in time of each limit state is shown from 0 years (thick line) to 7.01 years with an increment of 1 year. The corresponding MPPs are:

$$\mathbf{MPP}_1 = [-0.713, -1.09, 0.940] \quad \text{at } t = 7.01 \text{ years,}$$
$$\mathbf{MPP}_2 = [0.901, 1.387, -1.200] \quad \text{at } t = 0 \text{ years,}$$
$$\mathbf{MPP}_3 = [0.746, 1.151, -1.000] \quad \text{at } t = 0 \text{ years, and}$$
$$\mathbf{MPP}_4 = [0.800, 1.930, 2.170] \quad \text{at } t = 7.01 \text{ years}$$

with reliability indices $\beta_1 = 1.607$, $\beta_2 = 2.043$, $\beta_3 = 1.698$ and $\beta_4 = 3.012$.
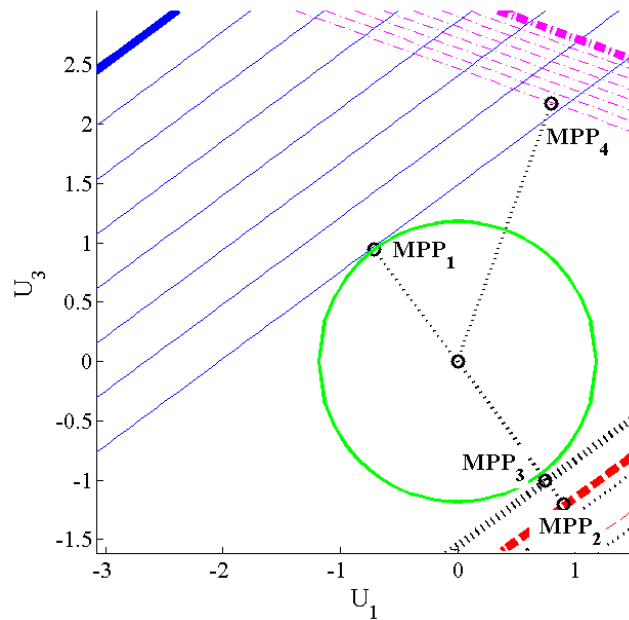

Figure 3. MPPs and limit states at the optimal design for a target reliability of 0.9 and a target cost of $18

As time progresses, the limit state $g_1 = 0.05 - \alpha \le 0$ expressing the minimum contact angle requirement, becomes more critical as it progressively approaches active status, and becomes active at 7.01 years (tangent to the target reliability circle at $\mathbf{MPP}_1$). On the other hand, the limit state $g_2 = \alpha - 0.17 \le 0$ expressing the maximum contact angle requirement, is more critical (closer to active status) at $t = 0$ years, attaining its $\mathbf{MPP}_2$ at 0 years. The limit state for the minimum torque requirement $g_3 = 3000 - \tau \le 0$ is more critical at $t = 0$ years and attains its

MPP$_3$ at 0 years. Finally, the limit state for the maximum hoop stress requirement $g_4 = \sigma_h - 400 \times 10^6 \leq 0$ becomes more critical and reaches its **MPP**$_4$ at 7.01 years.

Figs 4 through 6 show the optimization results for a target lifecycle cost C_L^t from $18 to $28 with an increment of $1. The target system reliability at the time for maintenance is equal to 0.9. For the initial design (dashed red line in Figs 4 and 5), the target reliability of 0.9 is reached at 3.37 years (Fig. 4) and the corresponding lifecycle cost is approximately $19 (see Fig. 5). For the initial design therefore, the time to maintenance is t_{0M} = 3.37 years. In contrast, for the optimal design corresponding to a target cost of C_L^t = $18 to $28, the optimal time to maintenance is t_M = 7.01 to 12.77 years which is much higher than 3.37 years. This is achieved by pushing the reliability curve at the optimal design to the right (blue lines in Fig. 4). Fig. 4 also shows that for a given time to maintenance, the reliability of the optimal design increases as the target lifecycle cost C_L^t increases.

The flat part of the blue lines in Fig. 5 indicates that the time to maintenance can be initially increased without any additional cost. This happens because the available cost is not spent on the non-critical requirements and is instead directed to the critical requirements. Thus, an optimal allocation of resources is achieved.

Although the maximum time to maintenance can be increased by increasing the target cost, it is ultimately limited by other constraints as shown in Figs. 4, 5 and 6. Thus, it cannot be further increased even if we provide more resources. Figs 5 and 6 clearly show that the constraint of the target cost remains active only until the target cost is raised to the level of approximately $26. For higher C_L^t values, the side constraints become active restricting the time to maintenance from increasing, even if more resources are provided.

This practical example clearly demonstrates that the time to maintenance can be optimized even if we have restricted resources.
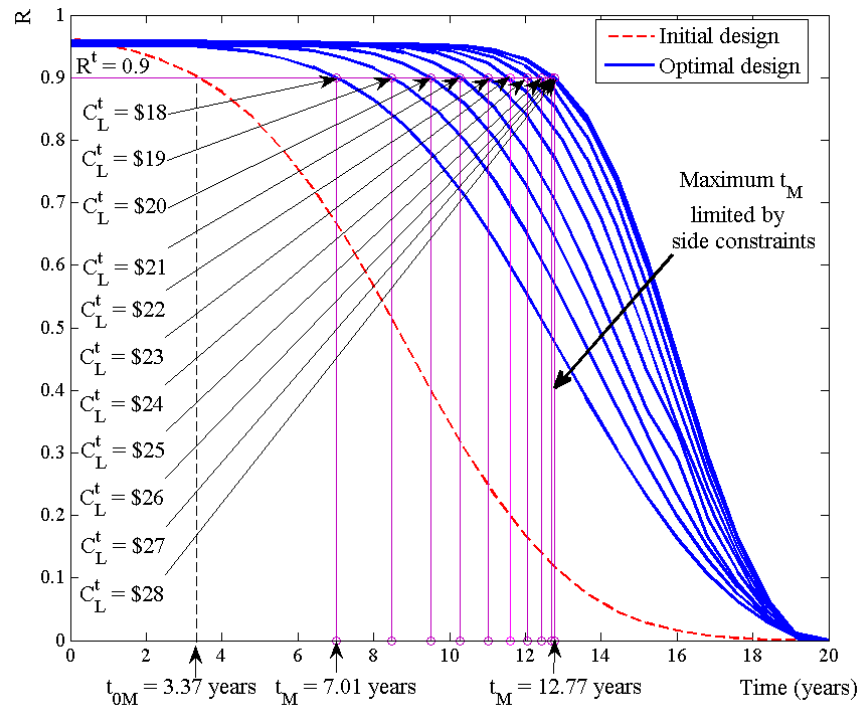
Figure 4. Reliability versus time and optimal time to maintenance for clutch example
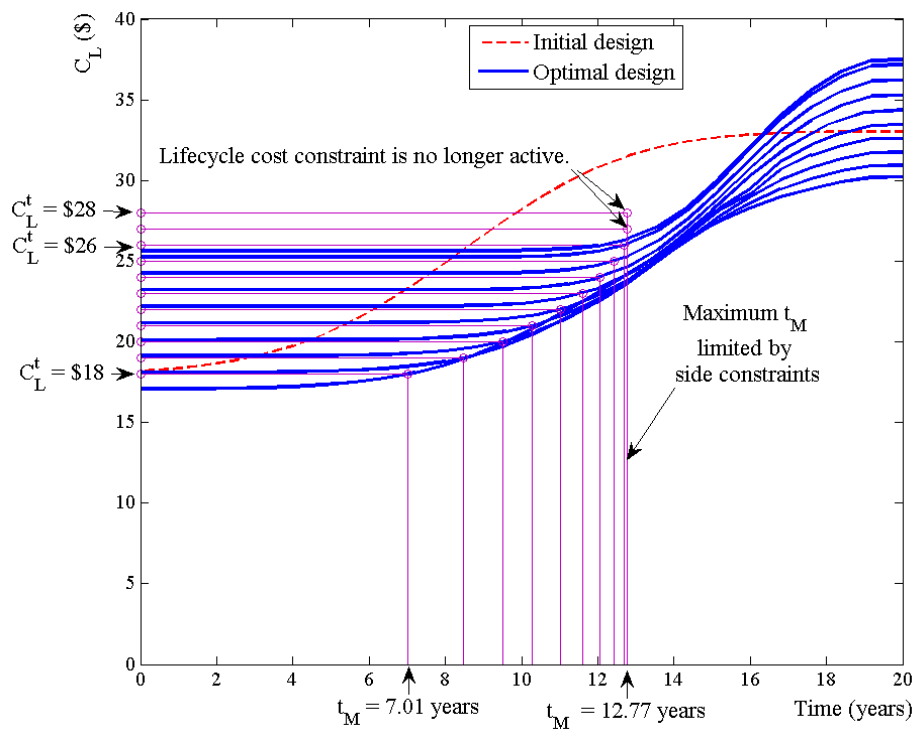

Figure 5. Cost versus time and optimal time to maintenance for clutch example
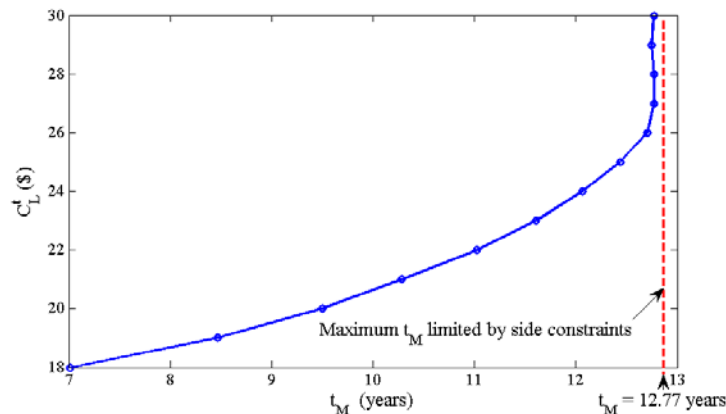
Figure 6. Cost versus time to maintenance for clutch example

6. SUMMARY AND CONCLUSIONS

Reliability is an important engineering requirement for consistently delivering acceptable product performance through time. As time progresses, the product may fail due to time phenomena such as time-dependent operating conditions, component degradation, etc. The degradation of reliability with time may increase the lifecycle cost due to potential warranty costs, repairs and loss of market share. In design for lifecycle cost, we must account for product quality, time-dependent reliability, and preventive maintenance. Quality is a measure of our confidence that the product conforms to specifications as it leaves the factory. While quality is time-independent, and reliability is time-dependent. Reliability depends on 1) the probability that the system will perform its intended function successfully for a specified interval of time (no hard failure), and 2) on the probability that the system response will not exceed an objectionable by the customer or operator, threshold for a certain time period (no soft failure).

For a repairable system, preventive maintenance is scheduled to avoid failures, unnecessary production loss and safety violations. We presented a methodology in this article, to increase the time to preventive maintenance while satisfying a minimum system reliability and a maximum allowable lifecycle cost. This was demonstrated using an automotive roller clutch example.

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